Principal Component Analysis: Computation of Latent Roots and Latent Vectors for Index Equation of Higher Secondary Schools

Ariba Shan[a,*] , Asif Ali Shaikh[a], Faheemullah Shaikh[a],Syed Feroz Shah[a], Habiba Khan[a]

[a]Mehran University of Engineering & Technology, Jamshoro, Pakistan.

Abstract

This article aims to analyze the performance of public-sector Higher secondary school education in Sindh, Pakistan. The Principal Component Analysis (PCA) methodology is implemented for the construction of two Higher Secondary School Educational Index (HSSEI) equations in the context of rural male (RM) and rural female (RF) for the period between 2000 and 2016 respectively, using fourteen essential school facilities that best contribute to the performance as indicators, and recommendations are provided accordingly. Results revealed that the highest contributing weights of identified indicators are such as 9.71% of Drinking water in RM and 9.663% of Institutions in RF. While the lowest contributing weights are such as 0.98% of Building condition in RM and 1.984% of Repeaters in RF. The examination of correlation of Enrolment and Repeaters with the rest of the indicators aided in identifying the significant interdependency among indicators and the measured weights of all indices provided a clear perspective towards upgrading the academic norms of Higher Secondary Schools.

Keywords: *Higher Secondary Schools; Principal Component Analysis (PCA) methodology; School infrastructure.*

1. Introduction

Education plays a meaningful role in human capital development. It is an interaction that empowers people to foster their life and lead a disciplined social life. It is one of the fundamental variables for improvement. The academic performance of all students is one of the imperative components, and due to such importance, the entire education system spins. All the educational sectors are held accountable for providing quality education and maintaining high academic standards for all students. While the quality of education cannot be judged yet, it can still be measured by specific performance indicators that can accelerate the requirements of education to move forward in a developing global environment in terms of social, economic, and cultural development [1].

Among other provinces of Pakistan, Sindh is the second most highly populated province and plays a vital role in the country's economy. According to [2], Pakistan economic survey in comparison with other provinces of Pakistan, the literacy rate of Sindh is deplorable that is in 2015-2016 it was 55% overall, 67% male, and 44% female, followed by urban areas 73% and rural areas 36%, where urban male stands by 80%, urban female 65%, rural male 51%, and rural female 19%. However, by the year 2019-2020, there is a slight increase in the literacy rate of Sindh that is overall 58%, urban 73%, and rural 39% [3]. As Higher Secondary School education is the final and the highest most crucial stage of school education in Pakistan, in this manner, it requires the utmost attention of educational administration to provide a productive and conducive learning environment and basic facilities for students and teachers because higher secondary schools are assumed to be the establishment towards higher knowledge in higher education. In Sindh, till now by 2019, 36% of schools lack from school boundary walls, 42% from drinking water, 20% from single classroom schools, 49% of schools managed by a single teacher, 9% from no permanent shelter, the post-primary schools are in dire straits, around 88.4% of system's functional schools are the Primary Schools, while just 6% provide middle-level education and functional higher secondary schools are only about 5.6% which is a disturbing circumstance. Else, out of school students have become a key concern for Sindh's education system, lack of or poorly equipped facilities, improper learning opportunities, poorly skilled teachers, overcrowding of students in a single classroom, and meager distribution of schools after every level is among the reasons for students who do not attend schools [4]. As stated by [5], Sindh occupies 39% of public sector higher secondary schools and more private institutions, which is 88%. But, the gender disparity proceeds to be a prominent issue at all levels of education along with inadequate availability of school infrastructure, including school buildings, electricity, boundary wall, toilets, and drinking water, in which toilets are critical to enhancing female' involvement in their participation at schools and lowering their dropouts. In contrast, only 30% of public sector school

*Corresponding author. Tel.: +92-336-3557929; fax: NA
E-mail address: ariba_arain@yahoo.com

buildings are satisfactory. Correspondingly higher secondary schools confront enormous challenges of receiving education in rural areas, which are somehow less progressed areas as compared to urban areas of Sindh, the need for teaching staff predominantly female teachers along with their qualifications, and student-teacher ratio leaves a very high impact on female's participation in rural areas schools. Rural areas endure more out-of-school children ratio than urban areas due to insufficient quantity of teachers along with vulnerable school conditions and several other restrictions limiting opportunities for females to get school enrolled.

The principal purpose of this paper is to ensure the educational policymakers that basic facilities accommodated to schools have a significant strong connection with substantial academic benchmarks because they are interdependent to achieve the advanced educational goals. Hence, there seems to be a gap in the literature of measuring the performance of indicators with developed weights as contributions of higher secondary schools of Sindh province. Principal Component Analysis (PCA) methodology is utilized to create the "Higher Secondary School Educational Index" (HSSEI) by using indicators that are supposed to be the most effective indicators for school education performance evaluation for constructing the index equations.

2. Literature Review

Extensive literature identifies different effective indicators that extremely influence all levels of school education in the rise and downfall of their performance by implementing various approaches. When school-provided facilities are in good condition and satisfactory, students and teachers are persuaded to do their duties to achieve the absolute outcomes of their performances. Limited necessary resources have remained a crisis for the public sector educational downfall in Sindh.

Educational amenities in schools provide the teaching and learning process a whole new significance such that they play an essential aspect in the accomplishment of a school's overall performance [6]. Distinct types of schools, specifically public and private, impact students' learning growth concerning their gender. The proportion of female students has been seen with a smaller amount of progress than male students, influenced by several indicators [7]. A study using the probit model and principal component analysis revealed that enrolment of students is influenced by regional wealth disparity, and rural areas become a target of high crisis, which can be adjusted by educational funds [8]. Another study using principal component analysis expressed that a robust positive correlation between school enrolment and school infrastructure has been observed in rural areas, signifying the importance of accessibility of school infrastructure with a significant focus on drinking water facilities and quantity of teachers in schools [9].

According to a study concluded by using the value-added teacher model, the urban-rural areas' school educational performances are solely dependent on the quality of teachers in terms of their qualification, whether academic or professional and the rural area's schools can be equivalent to urban areas schools' performances only if they can have the same qualified number of teachers [10]. Moreover, another study using Pearson product-moment correlation statistics found that the quality of education demands well-trained educated teachers. Still, a lack of professional development in teachers may affect students' achievements. By inferential and descriptive statistics, an uneven student-teacher ratio and teachers' qualification with their experience in teaching has a significant impact on students' performance and teachers' teaching competencies [11]. A study using principal component analysis attempted to determine the influence of infrastructure on enrolment by constructing a composite index of infrastructure. According to the findings, school physical infrastructure plays a crucial part in increasing the rate of schools' enrolment[12].

3. Data and methodology

An index is constructed by implementing a statistical technique of Principal Component Analysis (PCA) for two index equations, in this paper known as Higher Secondary School Educational Index (HSSEI) for public sector schools of Sindh, Pakistan. As stated by [13], PCA creates uncorrelated indicators, i.e., orthogonal known as Principal Components (PCs), which are a linear combination of original indicators. These linear combinations are the geometrical representation of the new coordinate system.

The index construction includes the following measures, i.e., correlation matrix, latent roots, latent vectors, rotated component loadings, and finally, the construction of weights of indicators. This study focus on data obtained from Pakistan Education Statistics 2000-2016, and the indicators which are selected, contribute more in an examination of higher secondary schools, i.e., x_1 = the total number of available institutions (INS), x_2 = the total number of enrolments in institutions (ENROL), x_3 = the total number of teachers in institutions (TCH), x_4 = the total number of professionally qualified teachers (TPQ), x_5 = the total number of academically qualified teachers (TAQ), x_6 = the number of student repeaters who repeat classes in institutions (RP), x_7 = the total number of available institution buildings (BL), x_8 = the total number of available electricity facility provided by institutions (EL), x_9 = the total number of available drinking water facility provided by institutions (DW), x_{10} = the total number of available washroom facility provided by institutions (WR), x_{11} = the total number of available boundary wall facility provided by institutions (BW), x_{12} = the total number of satisfied institution-building condition (BC), x_{13} = total student-teacher ratio in institutions (STR), and x_{14} = total teacher-institution ratio (TIR). The normalization of identified indicators is as follows, which scales down the indicators between zero and one.

$$\mathbf{Z}_{i,t} = \frac{x_{i,t} - \min(x_{i,T})}{\max(x_{i,T}) - \min(x_{i,T})} \tag{1}$$

$Z_{i,t}$ is the normalized indicator for i = 1 to 14 indicators, representing t as the corresponding year and T as a total observation period, i.e., 2000-2016, respectively. The correlation matrix is used to find the latent roots from the characteristic equation $|R - \lambda I| = 0$, where R is the correlation matrix, λ_i are the latent roots, I is the identity matrix. Hence, the calculated latent roots construct the rotated component matrix, which is the orthogonal rotations of latent vectors. Afterward, the coefficient of each indicator is formed by [14]:

$$\gamma_i = \frac{C_{ij} \ . \ \lambda_j}{\sum_{k=1}^{14} \lambda_k} \tag{2}$$

Where, γ_i represents the coefficient of each indicator, i = 1 to 14 (rows) for each indicator, j is the largest value of an extracted column of the rotated component matrix, so C_{ij} is the largest component value of i^{th} indicator (row) on j^{th} selected (column), λ_j is the latent root of j^{th} component, and λ_k is the sum of all latent roots. Moreover, the Kaiser (1960) rule is used to extract the principal components whose latent roots are greater than one. The final index weights of indicators are constructed as by [15].

$$(\mathbf{HSSEI})_\alpha = \frac{\sum_{i=1}^{14} X_i \ [\sum_{i=1}^{14} \gamma_i]}{\sum \ [\sum_{i=1}^{14} \gamma_i]} \tag{3}$$

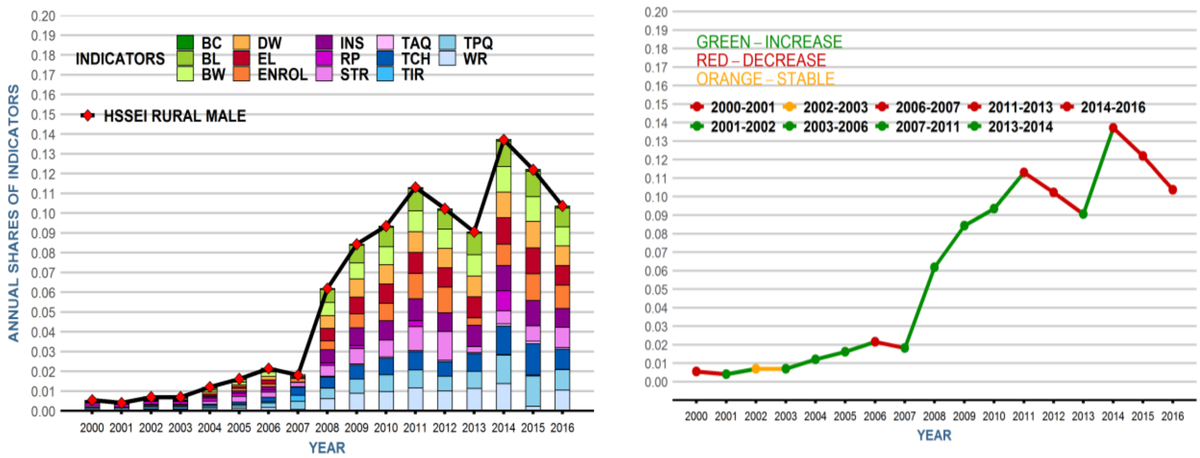
Where X_i is the i^{th} indicator and α defines the gender wise education i.e., Rural Male (RM) and Rural Female (RF).

4. Results and Discussions

4.1. Rural Male Model

$$(\mathbf{HSSEI})_{RM} = 0.09608\ INS + 0.09141\ ENROL + 0.09583\ TCH + 0.09583\ TPQ + 0.00985\ TAQ + 0.0262\ RP + 0.0962\ BL + 0.09694\ EL + 0.09707\ DW + 0.09132\ WR + 0.09676\ BW + 0.00982\ BC + 0.08641 STR + 0.01028 TIR. \tag{4}$$

From Table 2 of Rural Male Model, Enrolment strongly correlates with TCH, TPQ, STR, DW, EL, BW, BL, and WR, negatively strong with TIR, and only moderate with TAQ and BC, where remaining indicators are weak in relation. While the correlation of repeaters with all the remaining indicators is also weakly associated.



(a) (b)

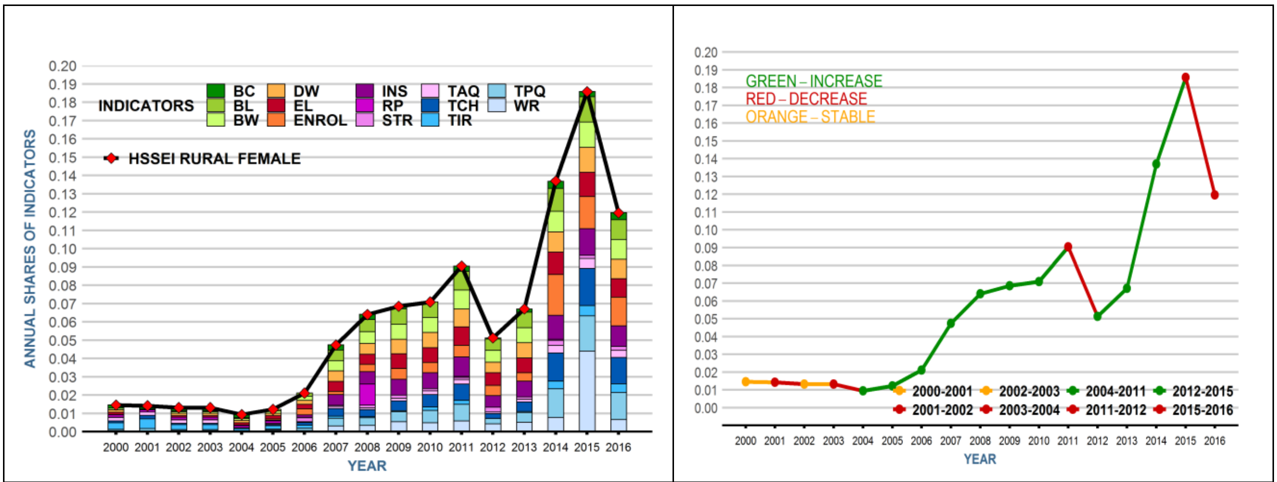
Fig.1. Rural Male (a) Contribution of Indicators 2000-2016 (b) Trends of HSSEI Rural Male

The final weights in Eq. (4) and the overall annual contribution of normalized indicators are further used to construct HSSEI of rural male per year historically. The composition of different trends is by the contribution annual shares of all indicators over the years in Fig.1 (a) The most affecting indicators, in 2000 and 2001 are RP (0.15%) ,TIR (0.11%), and in both years remaining indicators performed insignificantly, i.e., exactly, or nearer to (0%), where other indicators contribution are insignificant, in 2014, TCH and TPQ (1.44%), WR (1.39%), EL (1.33%), DW (1.3%), INS and BW (1.29%) with RP (1%), and in 2016, ENROL (1.18%), WR (1.05%), TPQ (1.04%), STR (1.03%), and TCH (1.02%).Thus HSSEI of the rural male is dominated by various high and minor shares of indicators for per year contribution in their performance.Fig.1 (b) represents different trends which are composed of several indicators shares, including four increasing trends 2001-2002, 2003-2006, 2007-2011, and 2013-2014, four decreasing trends 2000-2001, 2006-2007, 2011-2013, and 2014-2016, and only stable trend 2002-2003. During the beginning years, it is observed that very few indicators contributed and then gradually becomes progressing in further years. Thus, the overall performance of rural male has improved.

4.2. Rural Female Model

$$(HSSEI)_{RF} = 0.0966262\ INS + 0.0939727\ ENROL + 0.0933379\ TCH + 0.0933379\ TPQ + 0.0317527\ TAQ + 0.0198365\ RP + 0.095719\ BL + 0.0958645\ EL + 0.0963909\ DW + 0.0946477\ WR + 0.0964435\ BW + 0.0313679\ BC + 0.0204055\ STR + 0.0402971\ TIR. \quad (5)$$

From Table 2 of Rural Female Model, Enrolment strongly correlates with EL, BW, WR, DW, BL, TCH, TPQ, and STR, moderately correlated with TAQ, negatively moderate with RP, and other indicators are weakly correlated. Where repeaters have a moderate negative correlation with STR, WR, BW, BW, DW, and EL, the remaining indicators have a weak association.



(a)	(b)

Fig.2.Rural Female (a) Contribution of indicators 2000-2016 and (b) Trends of HSSEI Rural Female

The final weights in Eq. (5) and the overall annual contribution of normalized indicators are further used to construct HSSEI of rural female per year historically. The composition of different trends is by the contribution annual shares of all indicators over the year in Fig.2(a) The most affecting indicators, in 2000, TIR (0.36%), TAQ (0.19%), BC (0.18%), EL and DW (0.13%) with RP (0.12%), in 2004, BC (0.18%), EL and DW (0.13%) with RP and TIR (0.12%), while indicators including INS, TCH, TPQ, and TAQ contributed (0%), and remaining ones performed unsatisfactorily, in 2015, WR (4.40%), TCH (2.02%), TPQ (1.94%), ENROL (1.76%), INS (1.46%), BL, DW and BW (1.38%), and in 2016, ENROL (1.56%), TPQ (1.49%), TCH (1.44%), INS (1.12%) with RP (0.01%). Fig.2(b) represents different trends which are composed of various indicators shares, including two increasing trends 2004-2011, and 2012-2015, four decreasing trends 2001-2002, 2003-2004, 2011-2012, and 2015-2016, and only 2000-2001, and 2002-2003 are stable trends. So, the HSSEI of the rural female is influenced by various shares of indicators for per year contribution in their performance and same as in rural male, in the beginning, years it is observed that few indicators contributed with no significant enhancement in shares of indicators and then gradually becomes progressing in further years. Thus, the overall performance in the rural female has improved.

5. Comparative analysis

Table 5. HSSEI lowest to highest indicators weights in (%)

	$(HSSEI)_{RM}$	$(HSSEI)_{RF}$
1	0.98 BC	1.984 RP
2	0.99 TAQ	2.041 STR
3	1.03 TIR	3.137 BC
4	2.62 RP	3.175 TAQ
5	8.64 STR	4.030 TIR
6	9.13 WR	9.334 TPQ
7	9.14 ENROL	9.334 TCH
8	9.58 TPQ	9.397 ENROL
9	9.58 TCH	9.465 WR
10	9.61 INS	9.572 BL
11	9.62 BL	9.586 EL
12	9.68 BW	9.639 DW
13	9.69 EL	9.644 BW
14	9.71 DW	9.663 INS

Fig.3 compares the rural female and rural male indices of higher secondary schools constructed in this study. The highest peaks in rural female are in 2014 and 2015 by values of 0.137 and 0.1858, respectively, the lowest peak in 2004 by the value of 0.0094, where the highest peaks in rural male are in 2011 and 2014 by values of 0.1131 and 0.1371 and lowest peak in 2001 by values of 0.004. According to the contribution of indicators in rural female, ten increasing years occurred from 2004-2011 and 2012-2015, while four decreasing years from 2001-2002, 2003-2004, 2011-2012, and 2015-2016, where in rural male, nine increasing years occurred from 2001-2002, 2003-2006, 2007-2011, and 2013-2014 and six decreasing years occurred from 2000-2001, 2006-2007, 2011-2013, and 2014-2016. The frequent stable year in both rural female and rural male remained 2002-2003. Rural female has been at the highest shares of its all indicators in 2015 than rural male. In 2004, 2006, 2008, and 2014 whole contribution of indicators was very close enough of both indices, i.e., rural female by values of 0.0094, 0.0211, 0.064, and 0.137, rural male by values of 0.0121, 0.0215, 0.0619, and 0.1371. The significant continuous improvement in contribution of indicators also occurred in both indices, i.e., in rural female, 2004-2011 and 2012-2015, and in rural male, 2001-2006 and 2007-2011. However, both the indices since their early years struggled in their contribution and after 2004 begin to improve by participating more in their indicators shares.

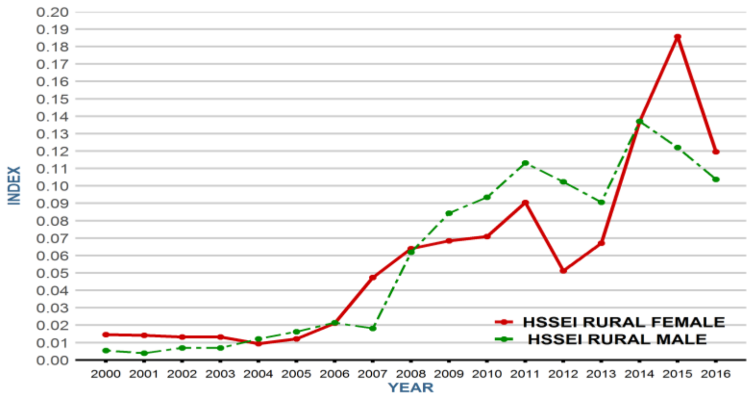


Fig.3. Comparison of HSSEI Rural Indices 2000-2016

6. Conclusion

This study constructed the public sector higher secondary school educational index (HSSEI) of Sindh, Pakistan. So, the two index equations concerning gender of rural areas are constructed by implementing PCA methodology and using fourteen basic school-provided facilities as indicators and obtained various weights of those indicators as a measurement of the contribution of their school performance for the period from 2000 to 2016. The highest contributing weights obtained for the indicators in Table.5 and the graphical interpretation of all index equations resulted such as: in rural male, DW, EL, BW, BL, INS, TCH, TPQ, ENROL, and WR, while its index improved after 2001 and in rural female, INS, BW, DW, EL, BL, WR, ENROL, TCH, and TPQ, while its index improved after 2004. By the examination of the correlation matrix of both rural male and rural female, the enrolment, and repeaters with the rest of the indicators, it is revealed that in those two indices, the enrolment was significant in both models implying that there is the direct interconnection among all the indicators.

7. Recommendation

This research analyzes the constructed HSSEI equations measuring the contribution of performance of indicators which is useful for educational policymakers to reformulate their policies according to the public sector higher secondary school education for provision of improved educational outcomes.

- Government should provide a sufficient educational budget keeping in view the gender wise institutions with an effective monitoring and evaluation mechanism which will transform the system into an improved educational environment with increased performance indicators' percentages.
- Since the student-teacher ratio is a crucial indicator in classroom learning, so a policy concerning the student-teacher ratio should be approved and implemented.
- All the less progressed dimensions of school physical infrastructure and teachers along with their qualifications are important attributes that cannot be disregarded in the light of the effects of globalization all over the world, hence it needs to be maintained for increasing enrolment rates.

Acknowledgement

The authors are wishing to acknowledge the Directorate of Post Graduate Studies Mehran University of Engineering and Technology, Jamshoro, Sindh, Pakistan and greatly acknowledge the provided comments, suggestions, and for their support to carry out this research.

References

[1] I. Ozturk, "The role of education in economic development : a theoretical perspective," *Munich Pers. RePEc Arch.*, vol. 2, no. 9023, pp. 22–29, 2018.
[2] "Pakistan Economic Survey 2019-20." [Online]. Available: https://www.finance.gov.pk/survey/chapter_20/PES_2019_20.pdf
[3] "Pakistan Economic Survey 2020-21." [Online]. Available: https://www.finance.gov.pk/survey/chapters_21/PES_2020_21.pdf
[4] "School Education Sector Plan And Roadmap for Sindh (2019-2024)," 2019, [Online]. Available: https://www.globalpartnership.org/sites/default/files/document/file/2020-19-Pakistan-Sindh-ESP.pdf
[5] *School education in Pakistan: A sector assessment*, no. June. 2019. [Online]. Available: https://www.adb.org/documents/pakistan-school-education-sector-assessment
[6] B.Nepal and R.Maharjan, "Effect of School's Physical Facilities on Learning and Outcomes of Students in Nepal," *J. Stud. Manag. Plan.*, vol. 1, no. 06, pp. 266–279, 2015, [Online]. Available: http://internationaljournalofresearch.org/index.php/JSMaP
[7] M. Catherine, "Gender Difference in Academic Achievement of Students in Kinangop Sub County, Nyandarua, Kenya," *Eur. J. Soc. Sci. Stud.*, vol. 5, no. 4, pp. 19–35, 2020. doi:10.46827/ejsss.v5i4.863
[8] H.N.Bashir, R.A.Khan, and M.Sajjad, "Regional Wealth Disparity and Its Impact on Urban and Rural Child Schooling in Developing Economies: A Case of Punjab (Pakistan)," *Review of Economics and Development Studies*, vol. 5, no. 4. pp. 615–622, 2019. doi: 10.26710/reads.v5i4.878
[9] J. Karmakar, "ASSESSING THE ENROLLMENT AND PRIMARY EDUCATIONAL INFRASTRUCTURE OF RURAL WEST BENGAL, INDIA: A DISTRICT LEVEL ANALYSIS," *J. S. Asian Stud.*, vol. 04, no. 03, pp. 101–108, 2016.
[10] J. Zhang, S.Jin, M.Torero, and T.Li, "Teachers and urban-rural gaps in educational outcomes," *Am. J. Agric. Econ.*, vol. 100, no. 4, pp. 1207–1223, 2018. doi: 10.1093/ajae/aay009
[11] A. O. Akinsolu, "Teachers and Students' Academic Performance in Nigerian Secondary Schools: Implications for Planning," *Florida J. Educ. Adm. Policy*, vol. 3, no. 2, pp. 86–103, 2010, [Online]. Available:

http://eric.ed.gov/?q=student+academic+performance&ft=on&id=EJ903008%5Cnhttp://files.eric.ed.gov/fulltext/EJ903008.pdf

[12] H.Majhi and M.Mallick, "Infrastructural Development and Enrollment in Elementary Education in Odisha," *Econ. Aff.*, vol. 64, no. 2, pp. 377–385, 2019 doi: 10.30954/0424-2513.2.2019.13

[13] I. T. Jolliffe and J. Cadima, "Principal component analysis: A review and recent developments," *Philos. Trans. R. Soc. A Math. Phys. Eng. Sci.*, vol. 374, no. 2065, 2016. doi: 10.1098/rsta.2015.0202

[14] I. Berk and V. S.Ediger, "A historical assessment of Turkey's natural gas import vulnerability," *Energy*, vol. 145, no. 2018, pp. 540–547, 2018, doi: 10.1016/j.energy.2018.01.022. doi:10.1016/j.energy.2018.01.022

[15] V. Ş. Ediger and I. Berk, "Crude oil import policy of Turkey: Historical analysis of determinants and implications since 1968," *Energy Policy*, vol. 39, no. 4, pp. 2132–2142, 2011. doi: 10.1016/j.enpol.2011.01.058

Appendix

A.1.Rural Male Model

Table 1. Summary Statistics

	INS	ENROL	TCH	TPQ	TAQ	RP	BL	EL	DW	WR	BW	BC	STR	TIR
Mean	0.44	0.40	0.36	0.36	0.43	0.15	0.44	0.43	0.43	0.39	0.44	0.53	0.35	0.2
Maximum	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Minimum	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Std.Dev.	0.41	0.39	0.32	0.33	0.27	0.23	0.40	0.39	0.39	0.36	0.38	0.26	0.31	0.25
Skewness	0.057	0.389	0.461	0.441	0.296	2.713	0.057	0.090	0.071	0.293	0.088	-0.146	0.591	1.807
Kurtosis	-1.893	-1.660	-1.060	-1.112	-0.307	7.200	-1.890	-1.846	-1.872	-1.761	-1.835	-0.296	-0.944	3.363
# Of Observations	17	17	17	17	17	17	17	17	17	17	17	17	17	17

Table 2. Correlation matrix (R).

	INS	ENROL	TCH	TPQ	TAQ	RP	BL	EL	DW	WR	BW	BC	STR	TIR
INS	1													
ENROL	.8649 ***	1												
TCH	.9428 ***	.9258 ***	1											
TPQ	.9428 ***	.9258 ***	1 ***	1										
TAQ	.686 ***	.6116 ***	.6983 ***	.6983 ***	1									
RP	-.1306	-.3395	-.2946	-.2946	-.1752	1								
BL	.9982 ***	.8698 ***	.9484 ***	.9484 ***	.6805 ***	-.14	1							
EL	.9816 ***	.8896 ***	.9662 ***	.9662 ***	.6642 ***	-.1969	.9834 ***	1						
DW	.9846 ***	.8993 ***	.976 ***	.976 ***	.6737 ***	-.2198	.9865 ***	.9945 ***	1					
WR	.906 ***	.8155	.8847 ***	.8847 ***	.5651 **	-.0714	.9078 ***	.9257 ***	.9072 ***	1				
BW	.9865 ***	.8796 ***	.9514 ***	.9514 ***	.6712 ***	-.1758	.9865 ***	.9975 ***	.9901 ***	.9219 ***	1			
BC	.6095 ***	.5451 **	.6078 ***	.6078 ***	.7952	-.1792	.6006 **	.6228 ***	.6242 ***	.5213 **	.6369 ***	1		
STR	.808	.9454 ***	.8366 ***	.8366 ***	.5015 **	-.2654	.8123	.8175	.8209	.8391 ***	.8123	.4615 *	1	
TIR	-.8026	-.7272	-.6992 ***	-.6992 ***	-.4112	.0416	-.7983	-.7849	-.7704	-.866 ***	-.8008	-.4647 *	-.8262 ***	1

Note: *,**,*** represents the statistical significance at 10%, 5%, and 1% level of significance.

Table 3. Latent roots evaluated from matrix R.

	λ_1	λ_2	λ_3	λ_4
Latent roots	10.8933	1.1091	0.9600	0.4267
Variance (%)	77.8096	7.9222	6.8569	3.0479
Cumulative Variance Explained (%)	77.8096	85.7318	92.5887	95.6367

Kaiser criterion (1960): proposed to extract latent roots greater than one**.**

Table 4. Rotated component matrix.

	C_1	C_2
INS	0.29581	0.10063
ENROL	0.28142	-0.08828
TCH	0.29504	-0.07650
TPQ	0.29504	-0.07650
TAQ	0.21787	-0.29780
RP	-0.07093	0.79214
BL	0.29618	0.09607
EL	0.29847	0.04851
DW	0.29886	0.01940
WR	0.28114	0.22525
BW	0.29791	0.06489
BC	0.20241	-0.29696
STR	0.26605	0.04963
TIR	-0.24704	-0.31090

Note: Two principal components extracted.

B.1. Rural Female Model

Table 1. Summary Statistics.

	INS	ENROL	TCH	TPQ	TAQ	RP	BL	EL	DW	WR	BW	BC	STR	TIR
Mean	0.39	0.25	0.27	0.28	0.34	0.10	0.41	0.43	0.41	0.13	0.41	0.48	0.42	0.42
Maximum	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Minimum	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Std.Dev.	0.35	0.30	0.30	0.31	0.27	0.24	0.34	0.33	0.31	0.23	0.32	0.32	0.31	0.31
Skewness	0.152	1.274	1.147	1.078	0.966	3.213	0.143	0.195	0.229	3.072	0.164	-0.049	0.240	0.520
Kurtosis	-1.557	0.359	0.060	-0.187	0.141	9.383	-1.559	-1.479	-1.421	8.760	-1.498	-1.064	-1.284	-1.108
# Of Observations	17	17	17	17	17	17	17	17	17	17	17	17	17	17

Table 2. Correlation matrix (R).

	INS	ENROL	TCH	TPQ	TAQ	RP	BL	EL	DW	WR	BW	BC	STR	TIR
INS	1													
ENROL	.9628 ***	1												
TCH	.9385 ***	.867 ***	1											
TPQ	.9385 ***	.867 ***	1 ***	1										
TAQ	.5532 **	.4587 *	.7169 ***	.7169 ***	1									
RP	-.6145 ***	-.6004 **	-.6158 ***	-.6158 ***	-.2346	1								
BL	.9937 ***	.9506 ***	.9264 ***	.9264 ***	.5379 **	-.6107 ***	1							
EL	.9838 ***	.9791 ***	.9057 ***	.9057 ***	.5099 **	-.6538 ***	.9683 ***	1						
DW	.9804 ***	.9522 ***	.9291 ***	.9291 ***	.5799 **	-.6518 ***	.9705 ***	.9844 ***	1					
WR	.9723 ***	.9579 ***	.889 ***	.889 ***	.5481 **	-.5952 **	.9625 ***	.9776 ***	.9666 ***	1				
BW	.9931 ***	.9734 ***	.9134 ***	.9134 ***	.537 **	-.6281 ***	.9838 ***	.9926 ***	.99 ***	.9831 ***	1			
BC	.3789	.3257	.475 *	.475 *	.5609 **	.221	.3765	.3474	.4122	.397	.3648	1		
STR	.7443	.8387 ***	.5945 **	.5945 **	.0888	-.5142 **	.7428	.7505	.7186 ***	.7002 ***	.7587	.0501	1	
TIR	.0713	-.0062	.2246	.2246	.7283	.2075	.0616	.0235	.0676	.0375	.0404	.4619 *	-.235	1

Note: *,**,*** represents the statistical significance at 10%, 5%, and 1% level of significance.

Table 3. Latent roots evaluated from matrix R.

	λ_1	λ_2	λ_3	λ_4
Latent roots	10.0901	2.2153	0.8098	0.4029
Variance (%)	72.0720	15.8237	5.7840	2.8781
Cumulative Variance Explained (%)	72.0720	87.8958	93.6797	96.5578

Kaiser criterion (1960): proposed to extract latent roots greater than one.

Table 4. Rotated component matrix.

	C_1	C_2
INS	0.31207	0.03089
ENROL	0.30350	0.09956
TCH	0.30145	-0.10276
TPQ	0.30145	-0.10276
TAQ	0.19265	-0.46709
RP	-0.20293	-0.29180
BL	0.30914	0.03616
EL	0.30961	0.06913
DW	0.31131	0.02053
WR	0.30568	0.03204
BW	0.31148	0.05248
BC	0.13007	-0.46143
STR	0.23110	0.30017
TIR	0.03643	-0.59278

Note: Two principal components extracted.